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On-Line Gain Identification of Flow Processes with Application to Adaptive pH Control

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A simple and practical method is presented for the control of first-order flow processes with time varying gain and a pure delay in the measurement of the control variable. It involves on-line identification of the process gain and a subsequent adjustment of the controller parameter. The method is well suited for applications in the chemical process industries where high frequency fluctuations in the process gain are not generally expected. It is tested on a computer simulated process as well as experimentally by application to a continuous stirred-tank neutralization process involving *p*H control, where buffer concentration in the feed varies with time. There is a potential for the application of this method to an industrial process such as wastewater treatment.

SCOPE

Processes whose gains vary widely during operation cannot be controlled satisfactorily by conventional fixed-parameter controllers. A solution to such a problem is to identify the process gain on-line and adjust the controller parameter suitably. This paper presents a method of on-line gain identification and control that applies to first-order flow processes with time-varying gain. An identification system (also of first order) in series with the process being controlled is perturbed by a signal consisting of a variable frequency, constant amplitude, rectangular wave to generate discrete estimates of the process gain. The identification is based on the fact that for a constant input to the identification system, the time taken for the identification system output to go from one preselected value to another is inversely proportional to the gain. The gain identification is achieved without disturbing the normal operation of the plant. The method is simpler and more easily implemented than the sinusoidal perturbation method proposed by Mellichamp et al. (1966).

The control strategy is based on maintaining a constant loop gain so as to maintain the same degree of stability. Thus, employing a two-mode (PI) or three-mode (PID) controller, the controller gain is self-adjusted as the process gain varies such that their product remains constant. All on-line computation, data storage, and control are handled by a moderate size analog computer with patchable logic.

The proposed method of identification and control is first tested on a computer-simulated process and then applied experimentally to a *p*H control system involving continuous stirred-tank neutralization of an acidic stream containing monobasic phosphoric acid as the buffer species by potassium hydroxide. This system can be modeled as a first-order process with time-varying gain. The variation in the process gain is caused by a change in the concentration of the buffer species in the feed stream. The perturbation input to the identification system consists of alternate flow of an acid (nitric acid) and a base (potassium hydroxide) at constant flow rate.

Process gain changes as large as 10 to 1 are introduced.

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CONCLUSIONS AND SIGNIFICANCE

The results of the tests performed to evaluate the proposed method of gain identification and control are quite satisfactory. During stationary (constant gain) operation, the gain estimate generated by the identifier equals the true process gain. During nonstationary operation, the identifier tracks the true gain satisfactorily, generating discrete estimates; the estimate lags the true gain of the identification system by a small margin and remains constant over a period of identification that decreases as the gain increases. The identification lag between the true process gain and the gain estimate primarily depends on the time constant of the identification system, the lag being smaller the lower the time constant. The method of identification has the inherent advantage of providing a more frequent identification as the process gain increases and the system tends to approach a less stable condition. It is especially suitable for applications in the chemical process industries, where high frequency fluctuations in the process gain are not generally expected.

A satisfactory control of the process is maintained despite a wide variation in the process gain. The experimental tests conducted to evaluate the proposed control strategy also yield satisfactory results and support the potential application of this method to an industrial process such as wastewater treatment involving *p*H control.

One of the control applications in the chemical process field involves control of processes whose gains vary widely during operation. Such processes are difficult to control by conventional feedback controllers. A solution to such a problem is to measure or identify the time-varying process gain and to use this information to adjust the controller parameter suitably. This method is termed adaptive control. The process gain cannot always be directly measured. It may be related to some unmeasurable state variable such as concentration. The gain will vary with time as the concentration varies.

This paper presents a simple and practical method for on-line gain identification and adaptive control applicable to first-order flow processes with time varying gain and a pure delay in the measurement of the control variable. One industrial application for which this method of identification and control may be well suited is the *p*H control of a process involving continuous neutralization of a plant waste stream. Among many applications of *p*H control, one that is assuming increasing importance is the treatment of plant effluent to meet the *p*H specifications required by local, state, or federal regulations. Such a system can be mathematically modeled as a first-order system with time-varying gain. The presence of weak acids, alkalis, or their salts in the plant stream leads to buffering. The process gain depends on the concentration of buffer species in the stream and can vary severalfold. Use of a conventional, fixed-gain, feedback controller tuned for the treatment of plant waste of a certain buffer concentration will often provide an unstable or unsatisfactory response when the buffer concentration changes. This can be prevented by the application of adaptive control.

A review of literature (Gupta, 1974) indicates that there is a lack of simple and practical techniques for on-line identification that do not require elaborate computational facilities and are easily implemented. Some of the proposed methods of identification require a well-defined test signal to be superimposed on the normal plant inputs and thus applied directly to the process being controlled, thereby disturbing the normal plant operation. Stochastic methods of identification such as correlation techniques using random test signals usually require too large an identification time to be practical. In the model reference or learning model approach to adaptive control, the parameter adjustment algorithms that would ensure convergence to the true parameter values at an acceptable rate of convergence tend to be computationally too involved.

Mellichamp et al. (1966) proposed a method for on-line gain identification applicable to first-order flow processes without disturbing the normal operation of the process. The identification was based on a modification of the sinusoidal perturbation technique in which the amplitude of the input sinusoidal signal was automatically adjusted to maintain a constant amplitude of the process output. Besides being somewhat complex, his method resulted in a significant lag in gain identification. The method proposed here is relatively simpler, easier to implement, and reduces the identification lag inherent in the method proposed by Mellichamp.

pH Control System

In order to better understand the general method of gain identification applicable to first-order flow processes, it will be appropriate to analyze a continuous stirred-tank neutralization process to which the method of identification and control will ultimately be applied.

A schematic representation of the process is shown in Figure 1. Consider that an acidic stream entering the tank at a volumetric flow rate G is being continuously neutralized by a strong base (the control reagent) entering at volumetric flow rate g. The *p*H of the tank effluent is the control variable which is to be maintained constant by manipulating the control reagent flow rate g. In an industrial application, the acidic stream may be a plant waste

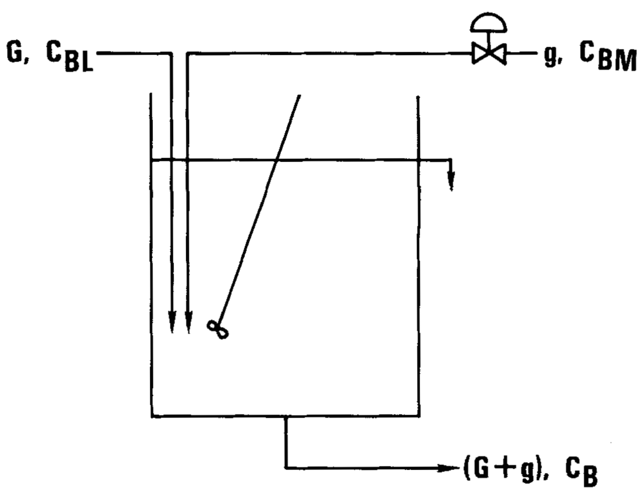

Fig. 1. Stirred-tank neutralization process.

stream containing weak acids or their salts as buffer species. The concentration of the buffer species constitutes the unmeasurable load variable which affects both, the tank effluent pH and the process gain. The acidic stream may also contain a strong base or acid, the concentration of which affects the pH of the tank effluent but has no effect on the process gain. Let C_{BL} be the concentration of strong base (or acid, which can be regarded as a negative base) in the acidic stream. Numerically, C_{BL} will be a positive quantity for strong base and negative for strong acid. Let C_{BM} be the concentration of strong base in the control reagent stream. Note that C_{BL} may vary, but C_{BM} will be kept constant. Also, for the purpose of this investigation the flow rate G of the acidic stream will be maintained constant.

Mathematical Model

In the pH control system under study, the acid base reactions are ionic and can be considered to take place instantaneously, with the result that the rate of reaction can be ignored. The stirred-tank process dynamics in this case would thus be similar to the case of mixing or blending nonreacting streams; the concentration of tank effluent would be governed only by the relative amounts of reagents added, and on the effectiveness of mixing.

If we assume perfect mixing and constant volume, a mass balance on the strong base yields

$$gC_{BM} + GC_{BL} - (G+g)C_B = V\frac{dC_B}{dt}$$

Dividing throughout by C_{BM} which normalizes all concentration variables with respect to C_{BM}, we get

$$V\frac{d\,c_B}{dt} + (G+g)c_B = g + Gc_{BL} \tag{1}$$

where

$$c_B = \frac{C_B}{C_{BM}} \quad \text{and} \quad c_{BL} = \frac{C_{BL}}{C_{BM}}$$

The term $(G+g)c_B$ is nonlinear. However, if G is very large compared to g, and variations in g are small, the following approximation will hold:

$$\begin{aligned} G+g &= G+\overline{g}+g^* \\ &\doteq G+\overline{g} \end{aligned} \tag{1a}$$

Combining Equations (1) and (1a), and dividing throughout by $(G+\overline{g})$, we get

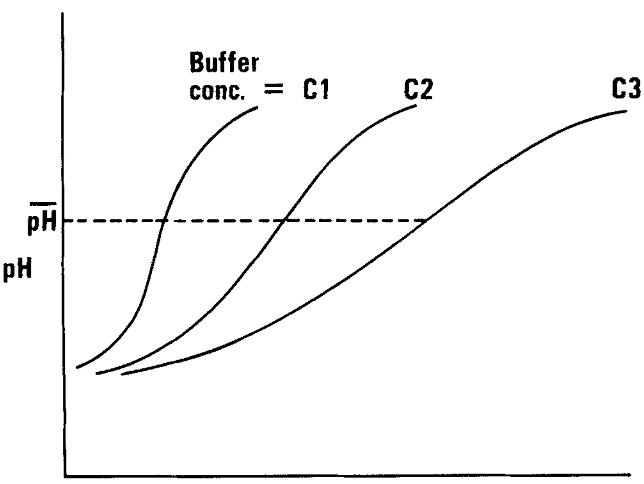

Fig. 2. Typical neutralization curves.

$$\frac{V}{(G+\overline{g})}\frac{dc_B}{dt} + c_B = \frac{1}{(G+\overline{g})}g + \frac{G}{(G+\overline{g})}c_{BL}$$

or

$$T\frac{dc_B}{dt} + c_B = K_M g + K_L\,c_{BL} \tag{2}$$

where

$$T = \frac{V}{(G+\overline{g})} \tag{2a}$$

$$K_M = \frac{1}{(G+\overline{g})} \tag{2b}$$

$$K_L = \frac{G}{(G+\overline{g})} \tag{2c}$$

At steady state, from Equation (2)

$$\overline{c}_B = K_M\overline{g} + K_L\,\overline{c}_{BL} \tag{3}$$

Subtracting Equation (3) from Equation (2), we get

$$T\frac{dc_B{}^*}{dt} + c_B{}^* = K_M g^* + K_L\,c_{BL}{}^* \tag{4}$$

Neutralization Curve

When a fixed volume of an acidic solution containing buffer species is titrated against a strong base of fixed strength, the resultant pH is a function not only of the amount of base added but also of the concentration of buffer species in the acidic solution. A plot of the amount of base added per unit total volume of the solution vs. the resultant pH, with buffer concentration as the parameter, is called the neutralization curve. For the flow system under study, c_B corresponds to the amount of base added per unit total volume of the solution, and the pH of the tank contents or tank effluent is the corresponding resultant pH. Typical neutralization curves are shown in Figure 2.

Consider a steady-state situation where the buffer concentration in the acidic feedstock is constant at C_2 and the effluent pH is at the desired value $\overline{p\text{H}}$. This is a stationary (constant buffer concentration) operation. If a plant disturbance causes the buffer concentration in the feed stream to change to C_1, the buffer concentration in the stirred tank would vary according to first-order dynamics and finally attain the new steady-state value C_1. Correspondingly, the neutralization curve for the system would change from the one corresponding to buffer concentration C_2 at the initial stationary state to that corresponding to C_1 at the final stationary state, moving through a continuum of curves corresponding to the instantaneous buffer concentrations in the stirred tank during the nonstationary operation.

Over a small region of interest around the operating point, where the neutralization curve can be taken to be linear, we get

$$p\text{H}^* = S\,c_B{}^* \tag{5}$$

During stationary operation, the slope S will be constant. Hence, from Equations (4) and (5)

$$T\frac{d\,p\text{H}^*}{dt} + p\text{H}^* = S(K_M g^* + K_L\,c_{BL}{}^*) \tag{6}$$

If we assume for the present that $c_{BL}{}^* = 0$ ($c_{BL}{}^*$ does not affect the process gain), Equation (6) reduces to

$$T\frac{d\,p\text{H}^*}{dt} + p\text{H}^* = S\,K_M g^* \tag{7}$$

When there is a time delay T_d in the measurement of the control variable pH^*, Equation (7) becomes

$$T \frac{d\,pH^*}{dt} + pH^* = S\,K_M g^*(t - T_d) \qquad (8)$$

This equation may be written in the general form

$$T \frac{dY_p}{dt} + Y_p = KX_p(t - T_d) \qquad (9)$$

where $K = SK_M$, the process gain which is proportional to S, K_M being a constant. Following a disturbance in the buffer concentration, the slope S will vary with a corresponding variation in the process gain K. This is nonstationary operation.

Gain Identification

The method proposed here for gain identification requires an identification system in series with the process being controlled and forcing it with a periodic perturbation signal. Physically, this is achieved by placing an identification tank in series with the process tank (Figure 3) such that it receives as feed a portion of the effluent from the process tank. This arrangement allows the identification of gain without disturbing the process. Furthermore, gain estimate is obtained at the operating point since the pH of the effluent from the process tank is at or near the set point.

The dynamics of the identification system, assuming there is no measurement lag, can be mathematically described by the equation

$$T' \frac{dY}{dt} + Y = K'X \qquad (10)$$

The proposed method of identification calls for a perturbation in X about the mean value zero according to a rectangular wave of constant amplitude A (Figure 4); X takes on a value of either $+A$ or $-A$. Consider the system to be initially at steady state when there is no input ($X = 0$) and $Y = 0$. At $t = t_0$, let X assume the value $+A$; as a result, Y will increase. At the instant Y reaches a preselected value Y_U, let X switch from $+A$ to $-A$, causing a decrease in Y. The instant Y reaches another preselected value $-Y_L$, let X switch back to $+A$ with a consequent rise in Y. The cycle will repeat with Y fluctuating between Y_U and $-Y_L$ and X switching between $+A$ and $-A$ as shown in the figure. An estimate of the gain is obtained according to the following analysis:

Integrating each term in Equation (10) over the time interval (t_0, t_1), we get

$$\int_{t_0}^{t_1} T' \frac{dY}{dt}\,dt + \int_{t_0}^{t_1} Y\,dt = \int_{t_0}^{t_1} K'X\,dt$$

During the stationary operation, K' is constant. Assuming that during nonstationary operation K' varies slowly so that it can be treated as a constant over the period of integration, and noting that $X = +A$ during this period, we get

$$T' \int_0^{Y_U} dY + \int_0^{T_0} Y\,dt = \hat{K}_1' \int_0^{T_0} A\,dt$$

or

$$T'Y_U + \int_0^{T_0} Y\,dt = \hat{K}_1' A\,T_0$$

where $\hat{K}_1'$ is the estimate of gain for the first period ($n = 1$) and is given by

$$\hat{K}_1' = \frac{T'Y_U + \int_0^{T_0} Y\,dt}{A\,T_0}$$

Similarly, for the time interval (t_1, t_2)

$$T' \int_{Y_U}^{-Y_L} dY + \int_0^{T_1} Y\,dt = \hat{K}_2' \int_0^{T_1} (-A)\,dt$$

which gives the gain estimate for the period $n = 2$

$$\hat{K}_2' = \frac{T'(Y_U + Y_L) - \int_0^{T_1} Y\,dt}{A\,T_1}$$

Likewise, for the period $n = 3$

$$\hat{K}_3' = \frac{T'(Y_U + Y_L) + \int_0^{T_2} Y\,dt}{A\,T_2}$$

When $Y_U = Y_L = Y_{UL}$, the gain estimate for the period n may be obtained from the general equation

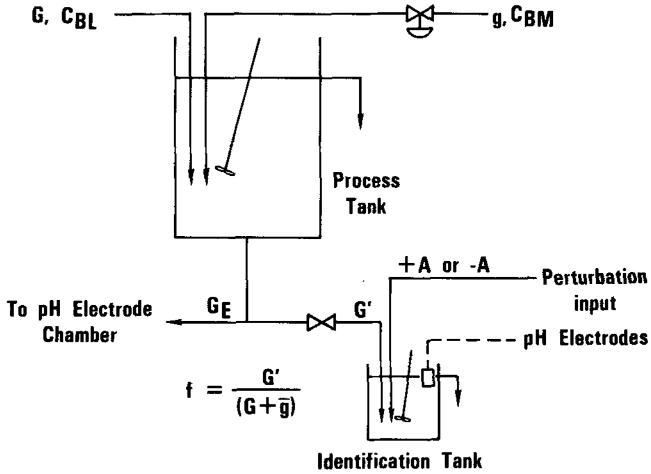

Fig. 3. Series arrangement of process and identification tanks.

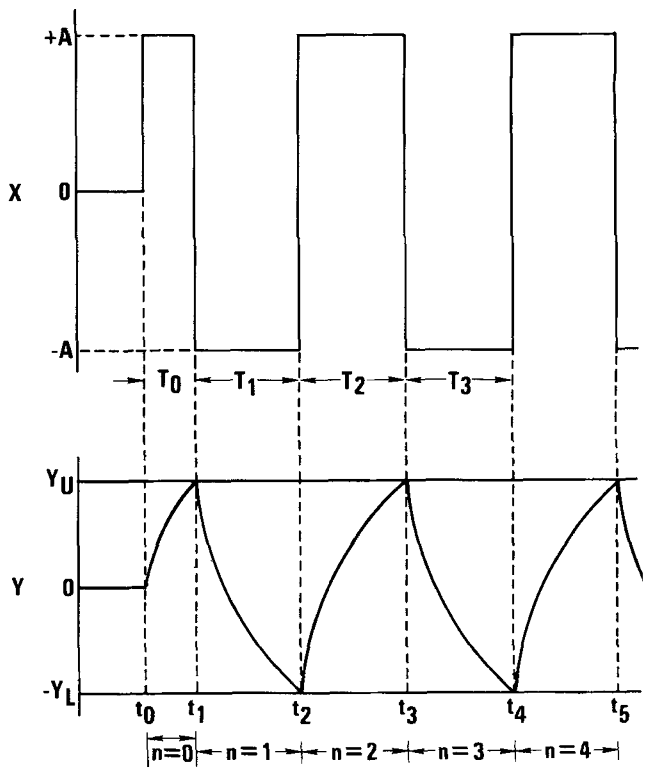

Fig. 4. Input output characteristics of the identification system.

$$\hat{K}_n' = \frac{2T'Y_{UL} + (-1)^{n-1} \int_0^{T_{n-1}} Y\,dt}{A\,T_{n-1}} \qquad (\text{for } n \neq 1) \tag{11}$$

$$= \frac{T'Y_{UL} + \int_0^{T_0} Y\,dt}{A\,T_0} \qquad (\text{for } n = 1) \tag{12}$$

The identifier, described mathematically by Equations (11) and (12), was implemented on an EAI 580 analog computer with patchable logic. The perturbation input X was generated using comparators and function relays in conjunction with logic components such as flip-flops and AND gates. The terms $\int_0^{T_{n-1}} Y\,dt$ and AT_{n-1} were generated by means of integrators, and the necessary data storage was carried out using the track-store units. Parameters T' and Y_{UL} being known, the on-line computation of the right-hand side of Equations (11) and (12) could be performed using standard linear and nonlinear components on the analog computer to obtain the gain estimate $\hat{K}_n'$. Figure 5 shows a simplified block diagram of the identification system.

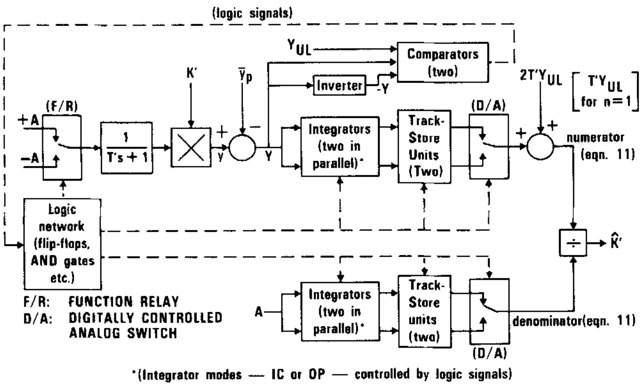

Fig. 5. Simplified block diagram of the identification system.

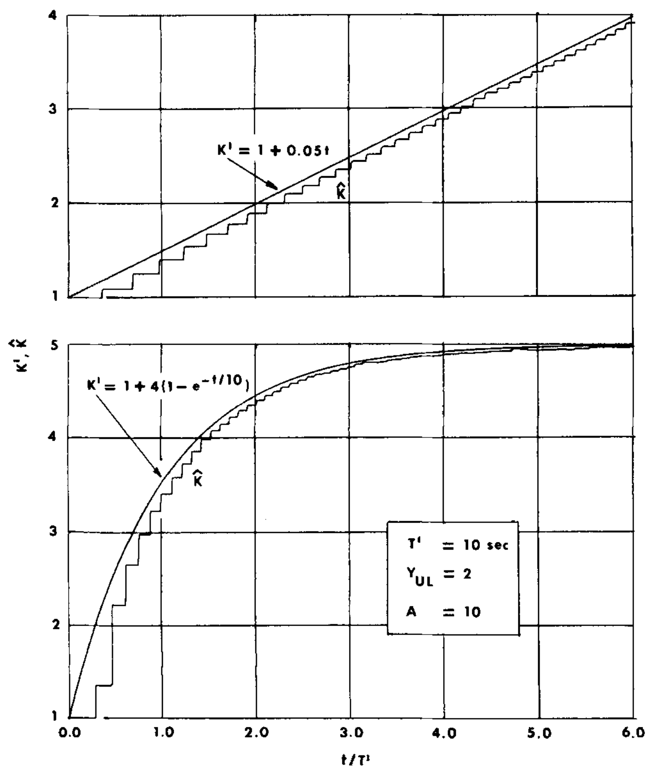

Fig. 6. Gain estimate for ramp and exponential variations in K'.

Application to a Computer-Simulated Process

The identification process represented by Equation (10) was simulated on the analog computer. To study the identifier characteristics, ramp and exponential variations in K' were introduced, and the gain estimate $\hat{K}'$ was generated according to Equations (11) and (12). The results are shown in Figure 6. It can be observed that the identifier tracks the true system gain K' satisfactorily, generating discrete estimates; the estimate lags the true value of the gain by a small margin and remains constant over a period of identification that decreases as the gain increases. During the stationary operation (observed in the case of exponential variation in K'), the gain estimate equals the true value of the gain, as expected.

Period of Identification

The period of identification T_n is given by

$$T_n = T' \ln \left[\frac{1 + Y_{UL}/A\,K_n'}{1 - Y_{UL}/A\,K_n'} \right] \tag{13}$$

The lower the time constant T' and the Y_{UL}/A ratio, the lower is the period T_n. Also, the higher the gain K_n', the lower would be the period T_n. If the ratio Y_{UL}/A is such that for the values of gain expected to be encountered

$$Y_{UL}/A\,K_n' >> 1$$

Equation (13) can be reduced to

$$T_n \doteq \frac{2T'}{K_n'}\,(Y_{UL}/A) \tag{14}$$

This equation is useful in the design of the identification system.

It may be noted that once the parameters T', Y_{UL}, and A are selected, the higher the gain, the higher the frequency at which the estimate of gain is generated. Since the system becomes less stable as the loop gain increases, the method of identification has the advantage of providing a more frequent identification as the system tends to approach a less stable condition. Also, the method is simple and easily implemented. For instance, compared to the sinusoidal perturbation method (Mellichamp et al., 1966), it is relatively easier to generate the rectangular wave of this method by means of comparators, relays, and logic components on the analog computer.

Applications in chemical process industries do not generally encounter a high frequency fluctuation in the process gain. The assumption of constant gain during a period of identification will not, therefore, introduce a significant error in gain identification. Also, the method is relatively insensitive to the plant noise; the integration involved in the method tends to average out any instantaneous error due to plant noise.

Identification Lag

Identification lag refers to the lag between the true process gain K and the gain estimate $\hat{K}$ generated by the identifier. For the pH control system under study, process gain K is defined in conjunction with Equation (9). Gain K' may be defined in a similar manner, and the relationship between K and K' may be expressed in the form

$$K = fK' \left(\frac{S}{S'} \right) \tag{15}$$

During the stationary operation, the buffer concentration in the identification tank will be the same as that in the process tank. Hence the neutralization curves for the solution in each tank will be identical, and the slopes at the operating point will be equal. Under this condition ($S = S'$)

$$K = fK' \tag{16}$$

The process gain estimate $\hat{K}$ is based on the estimate $\hat{K}'$ generated by the identifier and is given by

$$\hat{K}_n = f\hat{K}_n' \tag{17}$$

During nonstationary operation, the buffer concentration in the identification tank lags that in the process tank, causing S' to lag S. This results in a corresponding lag between K and $\hat{K}$ as evident from Equations (15), (16), and (17). A mathematical relationship between K and K' is given by

$$\frac{dK'}{dt} = \frac{1}{T'}\left(K' - f\frac{K'^2}{K}\right) \tag{18}$$

The above equation is derived (Gupta, 1974) on the basis of inverse relationship between buffer concentration and process gain, coupled with an unsteady-state mass balance on the buffer species entering and leaving the identification tank.

For the purpose of this study by computer simulation, f was taken to be equal to unity. This does not affect the generality of results. An exponential variation in K was introduced as indicated in Figure 7, for which a corresponding variation in K' was generated according to Equation (18). The gain estimate $\hat{K}$ was obtained from Equations (11), (12), and (17). To study the effect of T' on identification lag, two different values of T' were used. The values of other parameters such as Y_{UL} and A were the same for both runs, since these parameters do not affect the relationship between K and K'. It can be observed that a lower value of T' significantly improves the dynamics of the system and reduces the lag between K and $\hat{K}$.

Adaptive Control

A two-mode (PI) or three-mode (PID) conventional controller, well tuned for a constant-gain process, will provide unsatisfactory response when the process gain varies with time. A significant increase in the process gain will cause instability; on the other hand, the response will be too sluggish if there is a significant decrease in the gain. In order to maintain the same degree of stability despite variation in the process gain, adaptive control can be based on the simple strategy of maintaining the loop gain constant. Loop gain will be defined here as the product of controller gain and the process gain. Thus, a self-adaptive loop can be incorporated which automatically adjusts the controller gain as the process gain varies, such that their product remains constant. Figure 8 illustrates the control strategy.

Several methods for determining the optimum loop gain have been reported in the literature. They include the empirical method proposed by Ziegler and Nichols and methods based on error criteria such as minimum integral squared error (ISE), minimum integral time absolute error (ITAE), etc. In principle, any of these methods can be employed to establish the optimum loop gain. However, these methods are primarily meant for conventional, non-adaptive control systems and as such should be used only to obtain a first estimate of the desired loop gain. In the adaptive system under study, there are lags in the identification system which cause the actual loop gain to vary during the nonstationary operation. A more appropriate value of the loop gain setting can be determined only after observing the dynamic behavior of the system in response to a load disturbance.

Application to a Computer-Simulated Process

The adaptive control strategy was tested on a first-order process described by Equation (9) for which $T = 20$ s and $T_d = 2$ s. The process was simulated on the analog computer. The values of the identification system parameters were chosen to be $T' = 10$ s, $Y_{UL} = 2$, and $A = 10$.

Nature of the Load Disturbance

A change in the process gain constitutes the load disturbance. This can be explained with reference to Figure 9 which is a simplified representation of Figure 2. Note

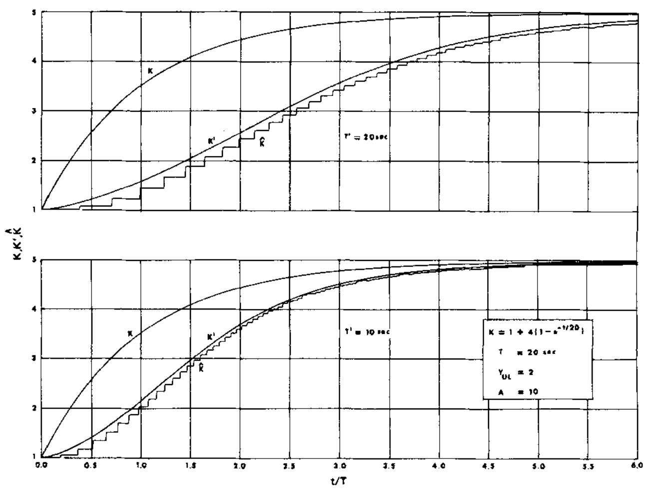

Fig. 7. Effect of T' on identification lag.

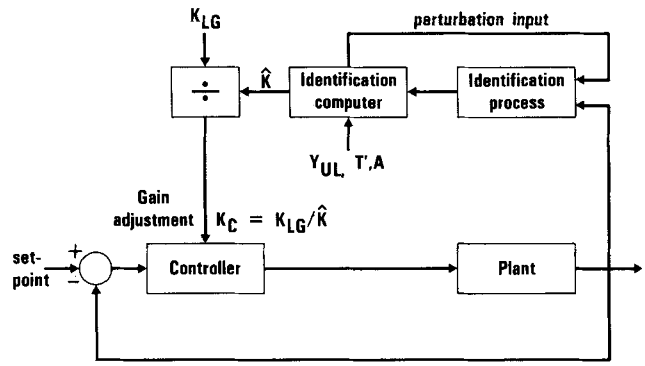

Fig. 8. Closed loop adaptive control system.

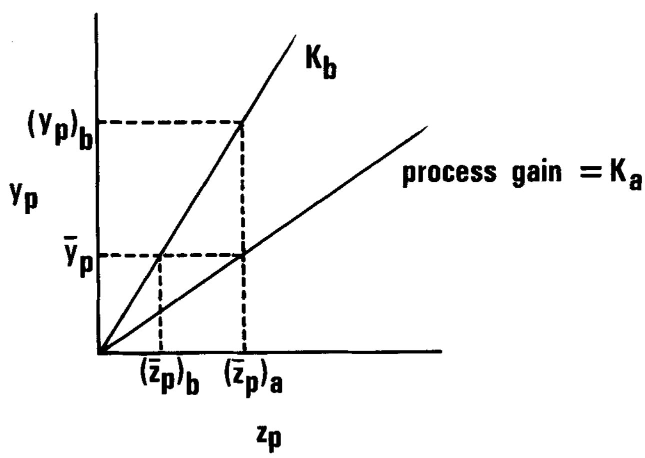

Fig. 9. Load change due to gain variation.

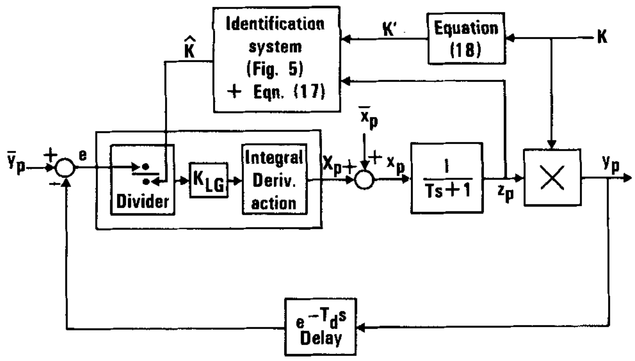

Fig. 10. Simplified block diagram of the adaptive control system (simulated process).

that y_p corresponds to pH, and z_p corresponds to c_B. Consider the process to be initially at steady state with gain K_a, $y_p = \overline{y_p}$, and $z_p = (\overline{z_p})_a$. Let there be a change in the variable affecting the process gain (for instance, buffer concentration in the case of pH control system) causing the process gain to vary and ultimately attain the value K_b. As a result, the control variable y_p will tend to move from its set point $\overline{y_p}$ and attain the value $(\overline{y_p})_b$. Any deviation in y_p from $\overline{y_p}$, however, will produce an error signal; the controller will manipulate x_p (corresponding to g) which in turn will adjust z_p until a new steady state is reached with $z_p = (\overline{z_p})_b$. For the experimental pH control studies to be discussed later, this load disturbance was introduced by a switch in the buffer concentration of the feed to the process, using solenoid valves.

Adaptive control performance of the computer-simulated process was studied for an exponential variation in the process gain described by the equation

$$K = 1 + 4(1 - e^{-t/20}) \tag{19}$$

Such a variation is considered to be fairly well representative of the type of gain variation expected to be encountered in an industrial application. A three-mode (PID) controller was implemented on the analog computer. The loop gain setting and other controller parameters (listed in Figure 11) were determined using the Ziegler-Nichols rules (Coughanowr and Koppel, 1965). A simplified block diagram of the adaptive control system is shown in Figure 10.

Discussion of Results

It is clear from the results (Figure 11) that a conventional, fixed-parameter controller is unsuitable for the control of a system such as the one under investigation. Note that, based on an initial process gain of unity, the controller gain K_C was set equal to the desired loop gain setting K_{LG} (for $K = 1$, $K_C = K_{LG}$). The temporary instability observed in the adaptive control response (for $K_{LG} = 9$) is due to the fact that during the transients, the actual loop gain increases significantly above the desired value of 9.0 as a result of identification lag. If necessary, this can be prevented by lowering the loop gain setting. There is scope for further study in the area of controller tuning applied to an adaptive control system such as this.

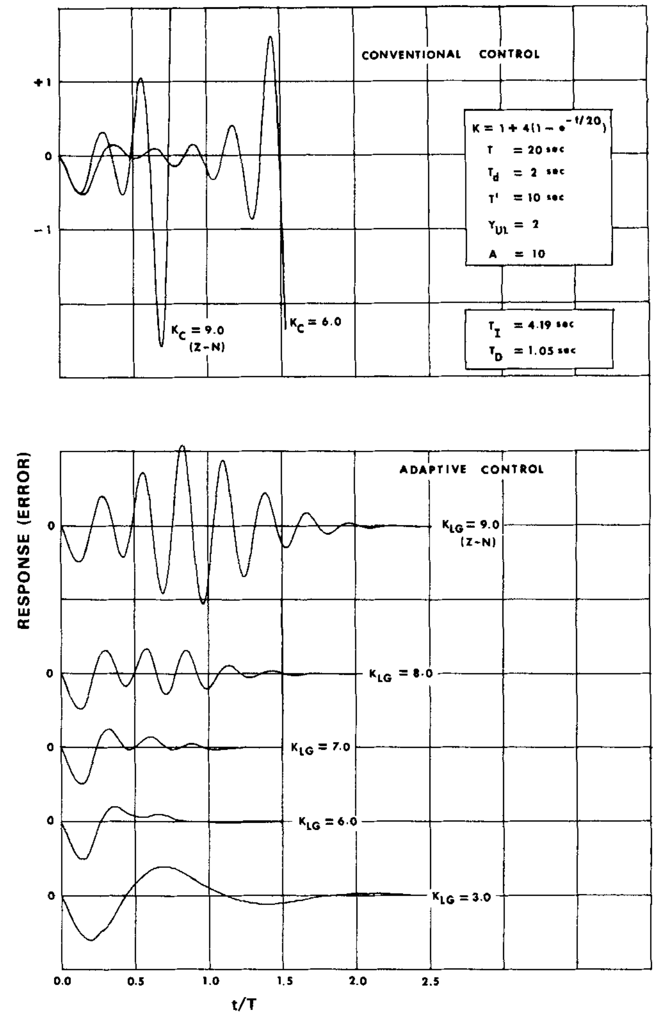

Fig. 11. Closed loop response to a load change for an exponential variation in K.

Application to pH Control

The method of on-line gain identification and adaptive control, tested earlier on a computer-simulated process, was applied to a real process involving continuous stirred-tank neutralization of an acidic stream by potassium hydroxide. The acidic stream consisted of an aqueous solution of monobasic phosphoric acid which is the salt of a weak acid and has a buffer property. In some of the test runs, the acidic stream also contained nitric acid.

Experimental Apparatus

Figure 12 shows the schematic of the pH control system. Acidic feedstocks or different buffer concentrations are prepared and stored in tanks T-1 and T-2. One of these feedstocks is initially fed at a constant rate of about 1 l/min from a constant-head tank (T-5, T-6) to the process tank T-9 with a holdup volume of about 3 l. It is neutralized by a controlled stream of 0.4M potassium hydroxide solution with a normal flow range of 30 to 170 ml/min supplied from a constant-head tank T-7 through a

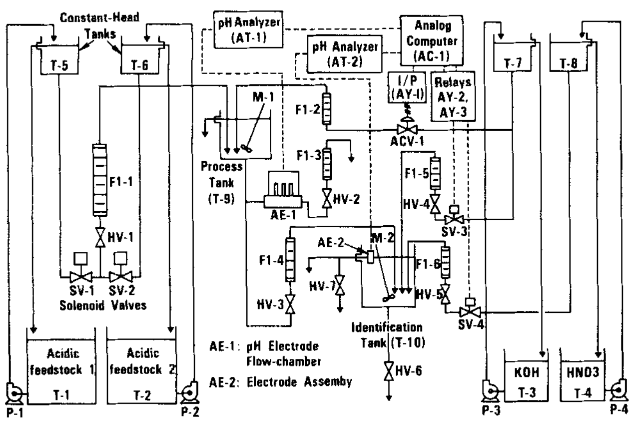

Fig. 12. Schematic of the pH control system.

TABLE 1. FEEDSTOCKS USED FOR CONTROL STUDIES

Feedstock	Composition	ml feedstock titrated against 0.4M KOH	Process gain 120K V/(ml/min)*
F1	0.10M KH_2PO_4	250	1.03
F2	0.06M KH_2PO_4	250	1.53
F3	0.02M KH_2PO_4	500	4.03
F4	0.02M KH_2PO_4 (500 ml) + 0.4M HNO_3 (40.2 ml)	500	4.99
F5	0.01M KH_2PO_4	1 000	7.79
F6	0.01M KH_2PO_4 (1000 ml) + 0.4M HNO_3 (93.5 ml)	1 000	10.24

* $K = SK_M$ V/(ml control reagent/min).
S = slope of the neutralization curve (Figure 13) at the operating point of pH 6.9 (corresponding to 2.9 V), V/(ml control reagent/ml total solution).
K_M [defined by Equation (2b)] $\doteq$ 1 000 (ml total solution/min)$^{-1}$.

pneumatic control valve ACV-1 ($C_V = 0.08$, linear trim). The pH of the tank effluent is measured using an electrode flow chamber assembly AE-1 in conjunction with a Beckman pH analyzer (transmitter) AT-1 which has an output signal range of 0 to 5 V corresponding to a linear pH scale of 4 to 9; thus, an output of 0 V corresponds to a pH of 4, and 5 V corresponds to a pH of 9. This output is compatible with the operating voltage range of the analog computer on which the controller is implemented. The controller output, a 1.1 to 5.5 V signal, is converted to a proportional 3 to 15 lb/in.[2] gauge pneumatic signal by means of an electropneumatic transducer AY-1 and sent to the control valve. Load disturbance in the process is introduced by switching the feed to the process from one feed tank to the other by means of solenoid valves SV-1 and SV-2. The flow rate of the tank effluent through the electrode chamber AE-1 is adjusted by means of manual valve HV-2 to get a certain predetermined time delay in the pH measurement.

A portion of the process tank effluent is fed at a constant rate of about 500 ml/min to the identification tank T-10 which has a holdup volume of about 1.5 l. The perturbation input, consisting of alternate flow of 0.4M potassium hydroxide and 0.4M nitric acid, is obtained by means of solenoid valves SV-3 and SV-4 which are actuated, through relays AY-2 and AY-3, by logic signals associated with the identifier circuit on the analog computer. These signals allow one solenoid to open at a time, admitting either potassium hydroxide or nitric acid at a constant flow rate of 100 ml/min adjusted by means of valves HV-4 and HV-5. The flow of potassium hydroxide corresponds to the input $+A$ (Figure 4), while that of nitric acid corresponds to $-A$. The pH of the solution in the identification tank is measured by means of dip electrodes AE-2 in conjunction with pH analyzer AT-2. This eliminates any measurement delay. The identifier on the computer receives a 0 to 5 V signal from the analyzer (corre-

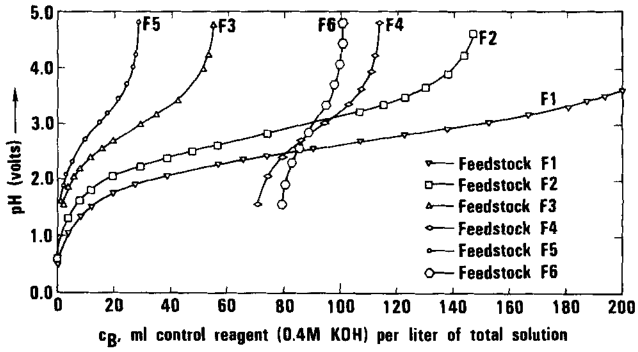

Fig. 13. Neutralization curves (experimentally determined).

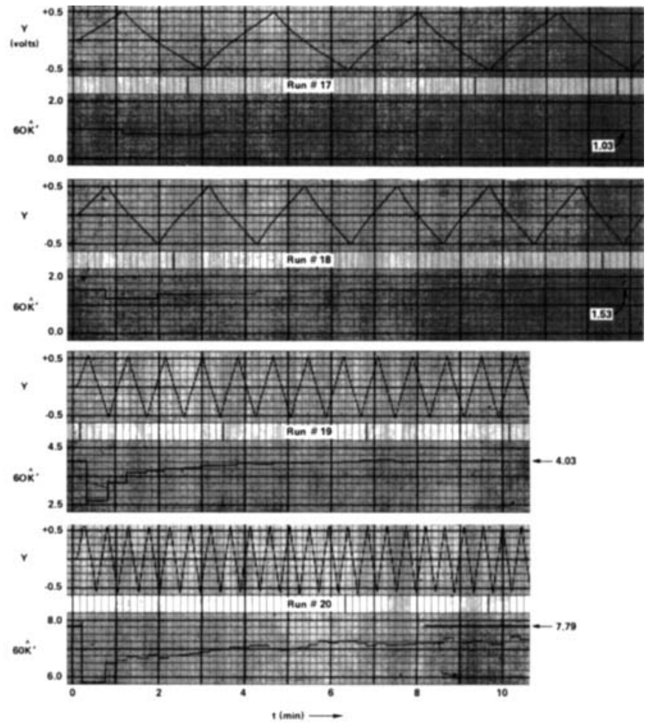

Fig. 14. Gain identification (stationary operation).

TABLE 2. RUN CONDITIONS AND RESULTS OF IDENTIFICATION TESTS

$Y_{UL} = 0.5$ V, $A = 100$ ml/min

KOH conc. = 0.4M, HNO_3 conc. = 0.4M

Feedstock flow rate = 498 ml/min

Run number	Feed-stock	Composition	Gain, 60 K' V/(ml/min)	Y_{UL}/AK'	T_n Equation (14), s	T_n Equation (13), s	T_n expt, s
17	A	0.1M KH_2PO_4 + 0.4M KOH @ 156 ml/l KH_2PO_4	1.03	0.2913	95.5	98.4	98.0
18	B	0.06M KH_2PO_4 + 0.4M KOH @ 90 ml/l KH_2PO_4	1.53	0.1961	64.3	65.2	64.0
19	C	0.02M KH_2PO_4 + 0.4M KOH @ 27.2 ml/l KH_2PO_4	4.03	0.0744	24.4	24.5	24.0
20	D	0.01M KH_2PO_4 + 0.4M KOH @ 13.6 ml/l KH_2PO_4	7.79	0.0385	12.6	12.6	13.5

sponding to a pH of 4 to 9) and generates a gain estimate according to the algorithm described earlier. All flow rates are measured by means of rotameters FI-1 through FI-6.

EXPERIMENTAL TESTING AND RESULTS

Preliminary Work

This mainly involved calibration of the six rotameters, obtaining characteristic curves for the electropneumatic transducer AY-1 (volts vs. pressure), and the pneumatic control valve ACV-1 (pressure vs. flow), check-out and standardization of pH analyzers AT-1 and AT-2 in conjunction with their respective electrode assembly AE-1 and AE-2, and obtaining neutralization curves for acidic feedstocks of different buffer concentrations (Table 1) used in this investigation. The neutralization curves are shown in Figure 13. The curves are fairly linear in the pH range of 6.35 to 7.40 (2.35 to 3.40 V).*

Dynamic Testing

To verify the dynamics of the process and identification system described by Equations (9) and (10), respectively, several test runs were conducted which involved introducing a step change in the potassium hydroxide flow through a solenoid valve, recording the pH response, and analyzing each run using the fraction incomplete response method. In the case of identification system, step changes were also introduced in the nitric acid flow. By an appropriate choice of feed composition and magnitude of step change, the pH response was confined to the linear range. All tests were conducted under flow conditions similar to those for subsequent identification and control studies.

The results indicated that the process could be characterized as being of first order with a time constant T of 164 s and with a measurement dead time T_d of 16 s. The dynamics of the identification system were similar to that of the process with a time constant T' of 164 s, but without any significant measurement delay.

Gain Identification

Process gain must be identified at the operating point which is the set point of the control variable. To test the method of identification experimentally, four feedstocks of different buffer concentrations (Table 2) were prepared, each with a pH of 6.9, being the operating point in all subsequent control studies. Under normal operation of the system with closed loop control, this would be the pH of the effluent from the process tank, a portion of which enters the identification tank. The slope of the neutralization curve at the operating point was determined for each of the feedstocks by titration against 0.4M potassium hydroxide and 0.4M nitric acid. The corresponding theoretical values of the process gain are listed in Table 2.

Stationary Operation. The identification method was first tested for stationary operation with each of the four feedstocks. The feed was directly admitted to the identification tank at a constant rate of 500 ml/min, and the system was perturbed with alternate flow of 0.4M potassium hydroxide and 0.4M nitric acid at 100 ml/min. The run conditions are listed in Table 2 and the results shown in Figure 14. The identifier performed quite satisfactorily, generating gain estimates equal or reasonably close to the true process gain. The actual period of identification compares well with that obtained from Equations (13) and (14). Note that the period of identification decreases with increase in the process gain.

Nonstationary Operation. To test the identifier for nonstationary operation, the feed was introduced at 500 ml/min directly into the identification tank through rotameter FI-1, and the identifier was put into operation. When steady state was attained, a disturbance affecting the process gain was introduced by a switch in the feed to the identification tank from one feed tank to the other using solenoid valves SV-1 and SV-2. The run conditions (Y_{UL}, A, feed composition, flow rates, etc.) were the same as those for stationary operation listed in Table 2. The results for the specified feed disturbances are shown in Figure 15.

At known time intervals during each run, samples were drawn through valve HV-7 and analyzed to obtain the true process gain at the time instants shown in Figure 15. Results indicate that the performance of the identifier during nonstationary operation is quite satisfactory. The frequency of identification increases as the process gain increases.

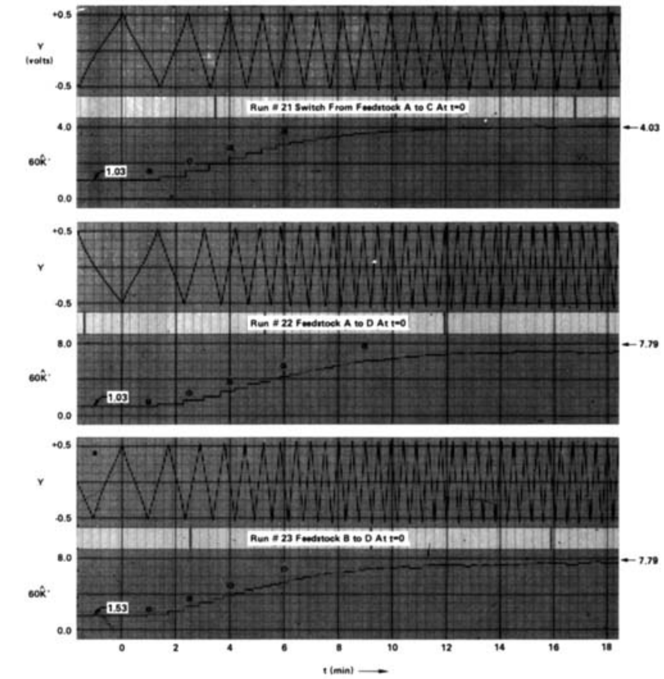

Fig. 15. Gain identification (nonstationary operation). Encircled points (⊙) indicate values determined by sample analysis.

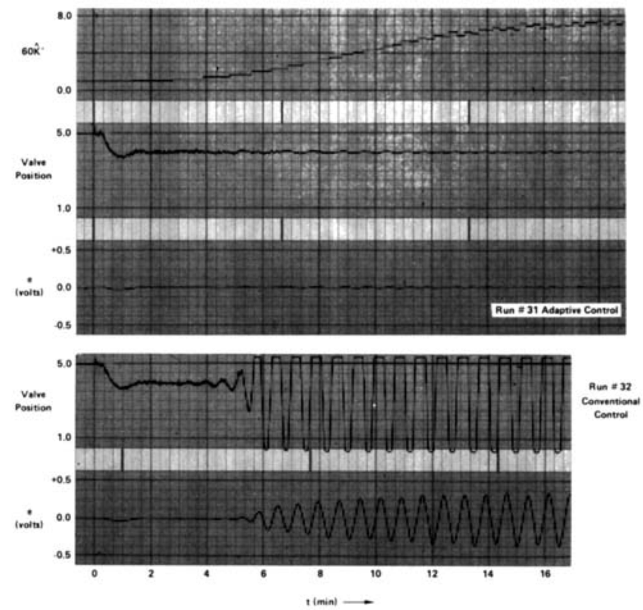

Fig. 16. Closed loop control (three-mode controller).

* Volts = pH − 4.00.

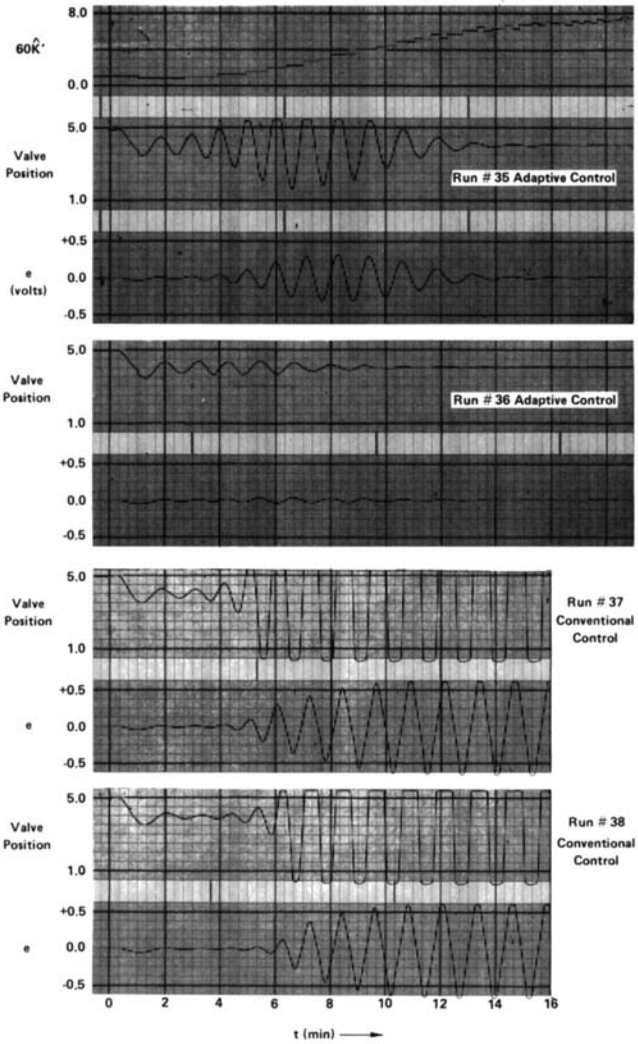

Fig. 17. Closed loop control (two-mode controller).

Closed Loop Control

Experimental evaluation of the proposed adaptive control strategy involved the operation of the entire system shown in Figure 12. The process was controlled by a PI or a PID controller implemented on the analog computer. Ziegler-Nichols controller settings were used. These settings were determined experimentally as well as by calculations based on known values of process parameters T and T_d, control valve gain K_V, and the transducer gain K_T (Gupta, 1974).

The initial feed to the process consisted of 0.1M monobasic phosphoric acid solution (feedstock F 1, Table 1) at 1 010 ml/min. When steady state was attained, the feed was switched to F6 through solenoid valves SV-1 and SV-2. The tank effluent pH (measured as the error signal e in volts), control valve position (input to the electropneumatic transducer), and the process gain estimate generated by the identifier were recorded on a strip-chart recorder. The response of the nonadaptive system to an identical disturbance under similar run conditions was also obtained. The results for different runs are shown in Figures 16 and 17. Table 3 summarizes the run conditions.

Discussion of Results

An error in the gain estimate generated by the identifier can result at higher process gain (run No. 20) due to the fact that the period of identification decreases at higher gains, and the gain estimate [Equation (11)] becomes more sensitive to errors in period of identification. Based on a knowledge of expected range of the process gain, too low a period of identification (less than 5% of the time constant T') should be avoided by a suitable choice of the parameters Y_{UL} and A [Equation (13)]. However, small errors in gain estimate can be ignored as long as the ultimate objective of satisfactory closed loop control is attained.

The results indicate that the proposed method of adaptive control is satisfactory for the control of flow processes with time-varying gain. The pH response to a load disturbance was excellent under the adaptive three-mode controller (run No. 31). In some instances (run No. 35), a temporary instability was observed for reasons discussed earlier (control of computer-simulated process). This may be avoided by lowering the loop gain setting (run No. 36) or by suitably retuning the controller. Also, a lower value of time constant T' with a corresponding decrease in the identification lag can help prevent this situation. It is interesting to note, however, that despite a relatively high value of T' in the present study ($T' = T$), satisfactory results were obtained, and there seems to be a potential for the application of this method to an industrial process.

TABLE 3. RUN CONDITIONS FOR CLOSED LOOP CONTROL

$Y_{UL} = 0.5$ V, $A = 100$ ml/min

KOH conc. = 0.4M, HNO_3 conc. = 0.4M

$G = 1\,010$ ml/min, $G' = 499$ ml/min, $G_E = 235$ ml/min

$T = 164$ s, $T' = 164$ s

Initial feedstock: F1 (Table 1)

Final feedstock: F6 (Table 1)*

Run number	pH set point	K_C (conv. control)	K_{LG} (adap. control)	T_I, s	T_D, s	$\bar{g}$ (initial), ml/min	$\bar{g}$ (final), ml/min
31	6.90	—	0.1818	30.83	7.71	149.7	92.5
32	6.91	21.19	—	30.83	7.71	148.3	Unstable
35	6.89	—	0.1674	47.50	—	144.8	95.0
36	6.89	—	0.1364	47.50	—	144.8	95.0
37	6.91	19.51	—	47.50	—	146.9	Unstable
38	6.92	15.89	—	47.50	—	148.3	Unstable

* The load disturbance introduced by switching the feedstock from F1 to F6 resulted in a tenfold increase in process gain.

ACKNOWLEDGMENT

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NOTATION

A = amplitude of the rectangular wave perturbation input to the identification system
C_B = concentration of base in the stirred-tank, g-moles/l
c_B = C_B normalized with respect to C_{BM}, volume base/volume total solution
C_{BL} = concentration of base in the acidic (load) feed stream, g-moles/l
c_{BL} = C_{BL} normalized with respect to C_{BM}, volume base/volume feedstock
e = error
f = $\dfrac{G'}{(G+\bar{g})}$
g = flow rate of the control reagent, ml/min
G = flow rate of the acidic feedstock to the process tank, ml/min
G' = flow rate of the stream from process tank to the identification tank, ml/min
G_E = flow rate of the stream through the pH electrode chamber (Figure 3), ml/min
K = process gain
K' = gain of the identification system
$\hat{K}$ = estimate of K
$\hat{K}'$ = estimate of K'
K_C = controller gain
K_L = constant, defined by Equation (2c), ml feedstock/ml total solution
K_{LG} = loop gain (product of process gain and controller gain)
K_M = constant, defined by Equation (2b), (ml total solution/min)$^{-1}$
K_n = process gain during the n^{th} period of identification
K_n' = gain of the identification system during the n^{th} period of identification
$\hat{K}_n$ = estimate of K for the n^{th} period of identification
$\hat{K}_n'$ = estimate of K' for the n^{th} period of identification
S = slope of the neutralization curve at the operating point, V/(ml base/ml total solution)
S' = slope of the neutralization curve at the operating point, for the contents of the identification tank
t = time, s
T = process time constant, s
T' = time constant of the identification system, s
T_d = time delay in the measurement of the control variable, s
T_D = derivative time of the controller, s
T_I = integral time of the controller, s
T_n = period of identification for the n^{th} period, s
V = volume of solution in the process tank, ml
X = perturbation input to the identification system
x_p = manipulated variable
X_p = manipulated variable, deviation from the steady state
y = identification system output
Y = identification system output, deviation from the steady state
y_p = control variable
Y_p = control variable, deviation from the steady state
Y_{UL} = value of Y corresponding to Y_U or Y_L (Figure 4) for $Y_U = Y_L$
z_p = variable corresponding to c_B for the process

Superscripts

$-$ = bar over a variable indicates its steady state value
$*$ = deviation from the steady state

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Velocity Distributions in Die Swell

Photographs of tracer particles were used to measure axial and transverse velocity components throughout the entire die swell region in a slit die. Laminar extrusion was investigated for two fluids: a viscoelastic concentrated solution of a polyacrylamide in glycerin and water and a highly viscous, nearly Newtonian, silicone oil. The die swell region extends upstream to the viscometric flow region and downstream to the relaxed portion of the extrudate. The data are used to evaluate several theories of die swell.

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SCOPE

When viscoelastic fluids such as polymer melts and solutions are extruded through an orifice or from a die at low Reynolds number, the extrudate often reaches a cross-sectional area several times that of the die opening. For circular dies, the extrudate diameter can become two to four times larger than the die diameter (Bagley, 1963; Vlachopoulos, 1972). The phenomenon is usually called die swell. Die swell is an important parameter for design of shaping dies used in polymer processing operations such as extrusion, blow molding, and injection molding. For noncircular die geometries, the extrudate not only changes dimensions, but it changes shapes as well.

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